The Dial-a-Ride Problem in Railways

Stephan Zieger [a,1], Nils Nießen [a],
[a] Institute of Transport Science, RWTH Aachen University
Mies-van-der-Rohe-Str. 1, 52074 Aachen, Germany
[1] E-mail: zieger@via.rwth-aachen.de, Phone: +49 (0) 241 80 25185

Abstract
The shift towards greater urbanisation in several countries has led to less emphasis being placed on railway lines in rural areas. In some cases, it is very difficult to sustain a railway service on these lines that is both demand-oriented and economically viable. As a result, services on many lines have been limited or even discontinued.
One way to make rural railway lines more attractive again and to operate them with an even stronger passenger-focus is to use small automated rail vehicles. This paper examines how an operation in such a scenario can be implemented. For this purpose, a mathematical optimisation program is presented which solves the resulting on-demand offline scheduling problem. Constraints such as time windows, vehicle capacities or occupation times of railway sections have to be taken into account.
The number of constraints quickly leads to large problem sizes, which can be reduced by pre-processing techniques – at least to some extent. The implementation and possible reductions of the problem are described in detail. The reductions have a moderate influence in a sensitivity analysis and can significantly contribute to enhance the solvability of the problem. The mathematical optimisation program is applied exemplarily to a regional line and yields an optimal schedule for smaller instances.

Keywords
Demand-Responsive Transport, Railway Operations, Scheduling, Automated Railway Units, Rural Area Railway Service

1 Introduction

The majority of the European Union's inhabitants now live in Cities, Towns or Suburbs (European Commission (2020)). This has often shifted the focus of railways to the maintenance and expansion of main lines, but rural areas are receiving less and less attention. Due to the reduced demand, regular railway operations have even been discontinued in some cases.

Especially in rural areas, it is very difficult to provide a demand-oriented, but also economically viable service with rail vehicles. The European Union has proclaimed the "Year of Rail" for 2021 (Council of the European Union (2020)) and would like to focus more on the operation of regional lines in future developments.

In recent decades, several disused railway lines have been reactivated or are to be reactivated, as there is often a certain passenger potential. There are reports on this from Italy (Corazza et al. (2020)), Spain (Eizaguirre-Iribar et al. (2015)), Portugal (Sarmento (2002)) and Germany (Verband Deutscher Verkehrsunternehmen e.V. (2020)).

According to Verband Deutscher Verkehrsunternehmen e.V. (2020), reactivation is supposed to fulfil a number of functions:

- development of underserved regions
- extension of passenger potential
- relief of current transport situation on road by shifting the traffic demand towards rail
- enhancing traffic connections between various origin-destination pairs
- improvement of current transport modes and/or substitution
- development of tourism by offering the service to reach certain destinations or by the extraordinary type of the service itself, e.g. steam locomotives or self-driving trains

In addition to the infrastructure-based solution of reactivation, operational measures can also be taken to offer services on lines with low demand economically. One possible solution is the use of smaller automated rail units that are not (exclusively) operated according to a fixed schedule, but are deployed according to demand. The use of such vehicles has a lot of potential, but there are also technical, operational and regulatory challenges. In this paper, the focus will be on the operational part, i.e. the setup of a feasible, and ideally optimal, schedule for serving passenger requests.

Demand-Responsive Transport (DRT) systems have existed in the road sector for a long time and the mathematical structure of such problems is well studied. The general task is to produce a schedule fulfilling the user requests for pick-up and drop-off while optimising some cost function and complying to several constraints, e.g, the number of available vehicles. Cordeau and Laporte (2003, 2007) give an overview of the fundamental models and general solution methods. There are many variations of the problem and they all try to capture real circumstances and improve them of which two are named exemplary. Posada et al. (2017) propose a model extension to incorporate fixed route public transport already in place into the flexible transport such that available resources are used more efficiently. Automated taxis are likely to be reality in the near future. Liang et al., 2020 research their behaviour in an urban environment and even include the travel time reduction in case too many such taxis are deployed. Ride-sharing does not only help to maximise profits, but also helps to reduce the overall amount of traffic.

To the best of the authors knowledge, Haverkamp's master thesis (Haverkamp (2017)), and the derived articles (Cats and Haverkamp (2018a,b)), are currently the only publications dealing with rail-bound DRT. They focus on a macroscopic level and try to facilitate direct transport on the main line network with passenger aggregation. One main result is that line capacity becomes much more important than station capacity – contrary to classic rail transportation.

In this paper we propose a Mixed-Integer-Program to solve the resulting on-demand offline scheduling problem. Several constraints such as minimum headway times, time windows for pick-up and drop-off, or capacity constraints for ride-sharing are included. The model builds on and fuses previous models from Cordeau and Laporte (2003, 2007) and Castillo et al. (2009, 2011).

The paper is organised as follows. In Section 2 the current states of railway transportation planning, automatisation of rail-bound vehicles and the setting of rural areas are assessed. Following, in Section 3 the mathematical programming model is presented and the corresponding implementation and pre-processing techniques are discussed. Section 4 gives a small computational example of application illustrating the capability of the model. Finally, Section 5 summarises the work and gives an outlook on open challenges and possible future research directions.

2 Initial Situation

From planning to the actual journey, several steps have been performed in canonical order up to now. However, the tasks change when traffic has to be planned in an on-demand fashion. The current procedure is first presented and then contrasted with the one for DRT. Afterwards, the technical classification of the required vehicles is briefly made, and the characteristics and chances in rural areas are discussed.

2.1 Classic Railway Transport Planning

An adequate and sustainable transport plan usually follows several stages as depicted in Fig. 1. All of these stages tend to be delicate on their own and are hence not performed in parallel, but in sequential order.

Figure 1: Classic transport planning concept (adapted from Goossens (2004))

In the first stage, a demand for the railway line has to be estimated. This is a crucial step for a sustainable transport concept and is often a show-stopper for smaller rural railway lines. There exist several different methods for the estimation of the demand which each have their unique advantages and disadvantages. The demand could be estimated, e.g. by the modelling with elastic functions which measure the inconvenience due to prolonged travel times (Rolle (1997)). The result of these demand estimation processes is usually an origin-destination matrix encoding the estimated amount of passengers travelling between two points. As the travel behaviour is usually fluctuating during the day a schedule incorporating these particularities should be in place.

Often, the different demands are aggregated in a line concept. The task is to set up lines covering the demands as good as possible, e.g. by minimising average travel times or the amount of changes. Railway line planning is already studied over a long time period and mathematical models are developed (Patz (1925); Bussieck (1998)).

The resulting railway lines are to be populated by train runs and therefore a schedule has to be designed. These specify the planned temporal and spatial movement of the trains and should fulfil certain properties, e.g. being conflict-free. In general, two types of schedules exist. First, the periodic schedule (Serafini and Ukovich (1989)) in which the train runs are repeated in a cyclic manner. Second, the non-periodic schedule which requires a higher planning effort but allows a higher flexibility.

In more detailed planning stages, the schedule needs further specifications. On one hand, the exact infrastructure occupation needs to be computed and resulting conflicts have to be resolved. On the other hand, the operators have to assign their vehicles to the schedule trips. Furthermore, the operators have to assign staff to these trips as well if the operation is not completely automated. Finally, infrastructure manager and operator have to include their maintenance and shunting movements in the plan.

All stages, but demand estimation, are resolving some allocation conflict with limited resources. Even for smaller networks, each of those stages is usually demanding and complex. Thus, the splitting approach enforces some restrictions just due to the order in which the stages are resolved. Depending on the stage, the individual tasks are performed in more or less detail more or less often. Therefore, Anthony (1968) proposed the concept of strategical, tactical and operational planning.

2.2 Demand-Responsive Transport

Demand-Responsive Transport (DRT) is a more recent transportation concept which can be defined by different means:

- ”DRT services provide transport on demand for passengers using fleets of vehicles scheduled to pick up and drop off people in accordance with their needs.” (Brand et al. (2004))
- ”DRT is a transport service where day-to-day operation is determined by the requirements of its users. Typically this involves users calling a booking service, which will then plan a route for the day to pick-up users and take them to their required destination.” (Enoch et al. (2004))

It originates from community transport and transport of people in need of (regular) medical care (Brake et al. (2006)). These people often cannot use public transport and need flexible solutions, e.g. for getting to their doctoral appointment. Another important use-case for DRT are rural municipalities in which standard public transport would not be feasible, but that want to offer a good coverage by public transport (Brake et al. (2006)).

DRT is a rather broad concept with many parameters as can be seen in Tab. 1. Interestingly, the concept is currently mainly used in road-bound transport.

The profile of a DRT service can be assembled from the non-complete table which results in a high number of different potential choices. Engels and Ambrosino (2004) list selected common scenarios and the authors come up with five general process steps:

1. The customer request is transmitted to the operator. The request contains information about the departure and arrival stop or address, the departure or arrival time, the number of required seats and, if necessary, any special requirements.
2. The customer is offered (several) preliminary possibilities with wide time windows (up to thirty minutes) for departure or arrival.
3. The time window is narrowed done to roughly five minutes when the date of travel is coming closer and a feasible schedule for this time period is computed.
4. The user confirms the booking and informs the operator of their intention to use the service offered.
5. The actual trip happens with the departure of the customer from his or her starting point.

Table 1: The different dimensions of DRT (summarised from König and Grippenkoven (2017); Brake and Nelson (2007); Hunkin and Krell (2018); Engels and Ambrosino (2004))

Route	Fixed route Fixed corridor Flexible route
Vehicle type	According to estimated demand
Operator	Commercial (Partly) subsidised public service
Origin-destination relationship	One-to-one One-to-many Many-to-one Many-to-many
Origin-destination service	Door-to-door Checkpoints
Booking by user	Telephone Internet (website/app)
Booking horizon	On-demand (shortly before) In advance (1 day+) Repeated occurrences
Service frequency	When requested Set number of journeys
Area coverage	Rural Suburbs Mixed
Payment	Pay on vehicle Season ticket SmartCard
Driver	Human Automated vehicle Autonomous vehicle
Operation Mode	Interchange DRT Network DRT Destination specific DRT Substitute DRT
User group	All public Disadvantaged groups Private groups
Payment	Free Paid
Competition	High Low

One particularly interesting variant is Dial-a-Ride (DAR). König and Grippenkoven (2017) defines DAR as operation without a schedule within a flexible area. The service is on-demand for all users within a pre-defined area. Booking can be done via telephone or internet. If DAR is combined with ride-sharing the efficacy of the system can be increased. Mathematically, the corresponding Dial-a-Ride Problem (DARP) attracted a lot of attention and the survey papers of Cordeau and Laporte (2003, 2007) can be consulted for an overview on different formulations and variants.

2.3 Highly Automated Vehicles

Modern vehicle technology allows for novel transportation concepts, too. One dimension is the interaction of the train with the infrastructure and external factors. The different variants are classified by the Grade of Automation (GoA) and are depicted in Fig. 2. The minimum requirement for Level 1 and following is the presence of Automatic Train Protection (ATP), i.e. a system which continually checks whether the speed requirements are fulfilled and which automatically stops a train violating those requirements. Levels 2 and following require an Automatic Train Operation (ATO) system. These systems fill the movement authorities generated with the actual train movement to a certain degree defined by the corresponding level.

For the intended service within this paper, vehicles that could be classified as GoA level 3 or better level 4 are necessary. Due to the nature of an on-demand service, the staff planning would hardly be able to be incorporated. One technical solution could be the use of a remote central instead of a train attendant for door closure and the operation in the event of disruption.

Grade of automation	Type of train operation	Setting train in motion	Stopping train	Door closure	Operation in event of disruption
GoA 1	ATP with driver	Driver	Driver	Driver	Driver
GoA 2	ATP and ATO with driver	Automatic	Automatic	Driver	Driver
GoA 3	Driverless	Automatic	Automatic	Train attendant	Train attendant
GoA 4	Unattended train operation	Automatic	Automatic	Automatic	Automatic

Figure 2: GoA scheme according to the International Association of Public Transport (Castells (2012))

Another dimension are the possibilities for flexible transportation concepts like DRT. The constant surveillance of and communication with the trains as well as the automation of various tasks enable transport concepts with a stronger passenger-demand focus. This is an advantage especially for less populated, usually rural, environments. The digitisation adds further value to the chain and lets the passenger communicate directly with the operator and thus the vehicles.

2.4 Characteristics and Chances for Rural Areas

In general, it is difficult for classic public railway transport to sustain a reasonable service level in a self-economic, or not heavily subsidised, manner in rural areas. The supply of public transport in cities is usually sufficient and well structured, but according to Eurostat (2020) only circa 40% of the European Union (EU) population lives directly in cities. Hence, some regions with low population density and demand which are exposed to infrequent public transport offer or without public transport at all could profit from reactivations of railway lines. Current concepts often lack the passenger focus, i.e. offering fix and infrequent rides. This is especially a shame as the overall appearance of the landscape is usually unimpaired and touristically worthwhile.

Currently, two trends seem to be present - on the one hand side the urbanisation for work, and on the other hand side the ruralisation for living due to space and living costs. If the public transport lacks adequacy people are more likely to use motorised individual transport with all of its consequences, e.g. traffic jams due to the traffic from outside the towns and cities. Besides these economical disadvantages, the ecological footprint of solo use of cars is not good. Last, the access to public transport has a social impact as well.

On one side, there is the described current stage with at least partially available disused railway tracks and a demand for transport, although today's public transport might not be able to appropriately satisfy it. On the other side, there are novel technological possibilities, e.g. automated small rail vehicles or intelligent digital communication. Furthermore, there exists a high share of potential passengers in rural areas which have no alternative (of using a car) or are limited by the restrictions. Therefore, there are good chances for implementing passenger-focussed urban rail transport, and ideally for intermodal integration as well.

3 The Dial-a-Ride Problem in Railways

The implementation of a DAR service in railways – Dial-a-Ride Problem in Railways (DARP-R) – faces several constraints. From an operational point of view, a feasible and ideally optimal schedule is to be set up for the on-demand operation. In the first part, a mathematical programming model is presented incorporating the most important constraints to be solved on a microscopic level. Afterwards, the implementation details for the model are described and, finally, pre-processing techniques to decrease the problem size are discussed.

3.1 Problem Formulation

The proposed formulation consists of two parts. The first part describes classic DARP constraints based on Cordeau and Laporte (2003, 2007). These constraints ensure the macroscopic vehicle flow and allocate most of the resources, but the exact track occupation. Due to different railway related restrictions, e.g. safety distances between train movements, the microscopic track assignment has to be incorporated. The related constraints are based on the work from Castillo et al. (2009, 2011).

The model aims to offer a high degree of flexibility in terms of adjustments, e.g. individual vehicle capacity or handling of similar requests by grouping. All parameters and variables relevant for the proposed model are summarised in Tab. 2.

The mathematical program reflects a service in which the users communicate their pick-up and drop-off point as well as the intended corresponding time windows. The operator collects these requests and tries to develop a schedule fulfilling constraints, e.g. vehicle capacity or headway time constraints, and with their available number of vehicles. They classically try to minimise their costs, i.e. an efficient allocation of resources and ride-sharing are necessary. Eq. (1)-(28) formalise the intended operational implementation. The explanation of the objective function and constraints follows below.

Table 2: Notation for the DARP-R

$[0, \bar{T}]$	Time window for operation
n	Number of requests
n_{stat}	Number of stations
$P = \{1, \ldots, n\}$	Pick-up locations
$D = \{n+1, \ldots, 2n\}$	Drop-off locations
$\{0, 2n+1\}$	Start and end vehicle depots
$\lvert A_{PD} \rvert = (2n(2n-1)-n)n_{stat}$	Auxiliary parameter
$H = \{3n+1, \ldots, 3n + \lvert A_{PD} \rvert + n_{stat}\}$	Intermediate vertices for headway constraints
$V = P \cup D \cup \{0, 2n+1\} \cup H$	Set of vertices in the graph
$A \subset V \times V$	Set of arcs
$G = (V, A)$	Directed graph on which the problem is defined
$(i, n+i)$	Request from i to $n+i$
c_{ij}	Travel cost for using arc (i, j)
t_{ij}	Travel time for using arc (i, j)
v_{ij}^k	Takes value -1, 0 or 1 depending on the travel direction of train k on arc (i, j)
q_i	Number of passengers at vertex i
d_i	Dwell time in vertex i
$[e_i, l_i]$	Time window of vertex i
L	Maximal ride time of passengers
K	Set of vehicles
Q^k	Capacity of vehicle k
T^k	Maximal operational duration for vehicle k
M_{ij}^k	Auxiliary variable for time tracking
W_{ij}^k	Auxiliary variable for load tracking
$Z_{ij}^{k_1,k_2}$	Auxiliary variable for train order tracking
w_i^k	Load of vehicle k upon leaving vertex i
r_i^k	Ride time for request $(i, n+i)$ in vehicle k
x_{ij}^k	Decision variable which is set to 1 iff vehicle k uses arc (i, j)
$h_{ij}^{k_1,k_2}$	Decision variable which is set to 1 iff vehicle k_1 uses arc (i, j) before vehicle k_2 does
arr_i^k	Arrival time at vertex i for vehicle k
dep_i^k	Departure time at vertex i for vehicle k

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \tag{1}$$

$$\min \sum_{k \in K} \sum_{i \in V} r_i^k \tag{2}$$

$$\min \sum_{k \in K} \sum_{(i,j) \in A} (arr_j^k - dep_i^k) \tag{3}$$

There are various minimisation functions that offer practical value. These are, e.g. the minimisation of the costs for all induced traffic (Eq. (1)), the minimisation of the sum over the individual travel times (Eq. (2)) or the minimisation of all travel times, including empty trips (Eq. (3)).

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1 \qquad \forall\, i \in P \tag{4}$$

$$\sum_{i \in V} x_{0i}^k = \sum_{i \in V} x_{i,2n+1}^k = 1 \qquad \forall\, k \in K \tag{5}$$

$$\sum_{j\in V} x_{ij}^k - \sum_{j\in V} x_{n+i,j}^k = 0 \qquad \forall\, i \in P, k \in K \tag{6}$$
$$\sum_{j\in V} x_{ji}^k - \sum_{j\in V} x_{ij}^k = 0 \qquad \forall\, i \in P \cup H \cup D, k \in K \tag{7}$$

Eq. (4)-(7) describe the general routing constraints. The vehicles have to start and end in a (virtual) depot, the same vehicle has to be used for the passenger collection at the pick-up and the delivery to the drop-off point. Furthermore, the vehicle entering and leaving a vertex must be the same.

$$arr_j^k \geq dep_i^k + t_{ij} - M_{ij}^k(1 - x_{ij}^k) \qquad \forall\, (i,j) \in A, k \in K \tag{8}$$
$$arr_{n+i}^k \geq dep_i^k \qquad \forall\, i \in P, k \in K \tag{9}$$
$$dep_i^k \geq arr_i^k + d_i \qquad \forall\, i \in V, k \in K \tag{10}$$

The preceding equations keep track of the time. Eq. (8) states that the arrival time at the following station must be at least as large as the departure time in the previous station plus the minimum travel time on that section. The subsequent constraint ensures time continuity, i.e. people being first picked up and afterwards dropped off. Last, Eq. (10) guarantees the drive-through or dwell time in a station.

$$w_j^k \geq (w_0^k + q_j)x_{0,j}^k \qquad \forall\, j \in P \cup \{2n+1\}, k \in K \tag{11}$$
$$\begin{aligned} w_j^k &\geq w_i^k + q_j - W_{ij}^k(1 - \sum_{k\in K} x_{ij}^k) \\ &+ (W_{ij}^k - q_i - q_j)\sum_{k\in K} x_{ji}^k \qquad \forall\, (i,j) \in A, k \in K \end{aligned} \tag{12}$$
$$w_{2n+1}^k \geq (w_i^k + q_{2n+1})x_{i,2n+1}^k \qquad \forall\, j \in D \cup \{0\}, k \in K \tag{13}$$

Similarly, the currently used capacity of the vehicles has to be tracked which is done by Eq. (11)-(13). Pick-up vertices have positive q_i, i.e. the number of passengers for this request, and the corresponding drop-off points have the proper capacity $q_{n+i} = -q_i$. As no pick-ups or drop-offs happen at the depot, $q_0 = q_{2n+1} = 0$ is set.

$$r_i^k \geq arr_{n+i}^k - (dep_i^k + d_i) \qquad \forall\, i \in P, k \in K \tag{14}$$
$$r_i^k \leq L \qquad \forall\, i \in P, k \in K \tag{15}$$
$$arr_{2n+1}^k - dep_0^k \leq T^k \qquad \forall\, k \in K \tag{16}$$
$$e_i \leq arr_i^k \leq l_i \qquad \forall\, i \in V, k \in K \tag{17}$$
$$\max\{0, q_i\} \leq w_i^k \leq \min\{Q^k, Q^k + q_i\} \qquad \forall\, i \in V, k \in K \tag{18}$$

The ride time of passengers is calculated in Eq. (14) as the time difference between pick-up and corresponding dwell time and drop-off time. It is constrained by a maximum ride time in Eq. (15) which could also be replaced by the shortest path increased by some detour factor for each individual request. Further constraints are on the maximum operating time of the vehicles (Eq. (16)), on the time windows for each vertex of the graph (Eq. (17)) and on the load of the vehicles (Eq. (18)). The current utilisation of the vehicle has to be between 0 and the vehicle capacity Q^k.

$$dep_i^{k_1} \geq dep_i^{k_2} + h_{ij,s} - Z_{ij}^{k_1,k_2} h_{ij}^{k_1,k_2} \tag{19}$$
$$dep_i^{k_2} \geq dep_i^{k_1} + h_{ij,s} - Z_{ij}^{k_1,k_2} (1 - h_{ij}^{k_1,k_2}) \tag{20}$$
$$arr_j^{k_1} \geq arr_j^{k_2} + h_{ij,s} - Z_{ij}^{k_1,k_2} h_{ij}^{k_1,k_2} \tag{21}$$
$$arr_j^{k_2} \geq arr_j^{k_1} + h_{ij,s} - Z_{ij}^{k_1,k_2} (1 - h_{ij}^{k_1,k_2}) \tag{22}$$
$$\forall\, i, j \in H, k_1, k_2 \in K, v_{ij}^{k_1} v_{ij}^{k_2} > 0$$
$$dep_i^{k_1} \geq arr_j^{k_2} - Z_{ij}^{k_1,k_2} h_{ij}^{k_1,k_2} \tag{23}$$
$$dep_i^{k_2} \geq arr_j^{k_1} - Z_{ij}^{k_1,k_2} (1 - h_{ij}^{k_1,k_2}) \tag{24}$$
$$\forall\, i, j \in H, k_1, k_2 \in K, v_{ij}^{k_1} v_{ij}^{k_2} < 0$$

Eq. (19)-(22) formalise the headway time constraints for trains travelling in the same direction. At first, it has to be decided whether train k_1 or k_2 is the leading train entering the section. Then, the trains have to be separated by a technical minimum amount of time – the headway time. For trains moving in opposing directions (Eq. (23) and (24)), either train has to move first. This constraint assumes two tracks per station for crossing. If the infrastructure is not available, the headway constraints for opposing trains have to be extended to include adjacent sections. During the occupation of the first train, the whole section is then blocked for the second train.

$$x_{ij}^k = 0 \text{ or } 1 \qquad \forall\, (i, j) \in A, k \in K \tag{25}$$
$$h_{ij}^{k_1,k_2} = 0 \text{ or } 1 \qquad \forall\, i, j \in P \cup H \cup D, k_1, k_2 \in K \tag{26}$$
$$0 \leq arr_i^k \leq \bar{T} \qquad \forall\, i \in V, k \in K \tag{27}$$
$$0 \leq dep_i^k \leq \bar{T} \qquad \forall\, i \in V, k \in K \tag{28}$$

Finally, the two decision variables x_{ij}^k (Eq. (25)) for arc ussage and $h_{ij}^{k_1,k_2}$ (Eq. (26)) for train precedence as well as arr_i^k (Eq. (27)) for arrival time and dep_i^k (Eq. (28)) for departure time in each vertex are defined.

3.2 Implementation of the Model

MATLAB (2020) serves as the framework for the implementation. The general workflow of the program is depicted in Fig. 3.

At the beginning, the necessary parameters for the time horizon ($\bar{T}$), number of requests (n), number of railway stations (n_{stat}) and vehicle capacity (Q^k) have to be defined. Furthermore, the track layout has to be determined and the corresponding travel times t_{ij} have to be calculated for each section (i, j).

In the second stage, the requests are created. The origin-destination pairs are generated by a uniform distribution or (two overlapping) normal distributions simulating one (two) major stations with more demand/supply on the underlying infrastructure. The second setup would correspond to one or two larger towns/cities which are usually the end-points of rural railway lines. Within the generation process, a random number $0 < q_i \leq \max_k\{Q^k\}$ is

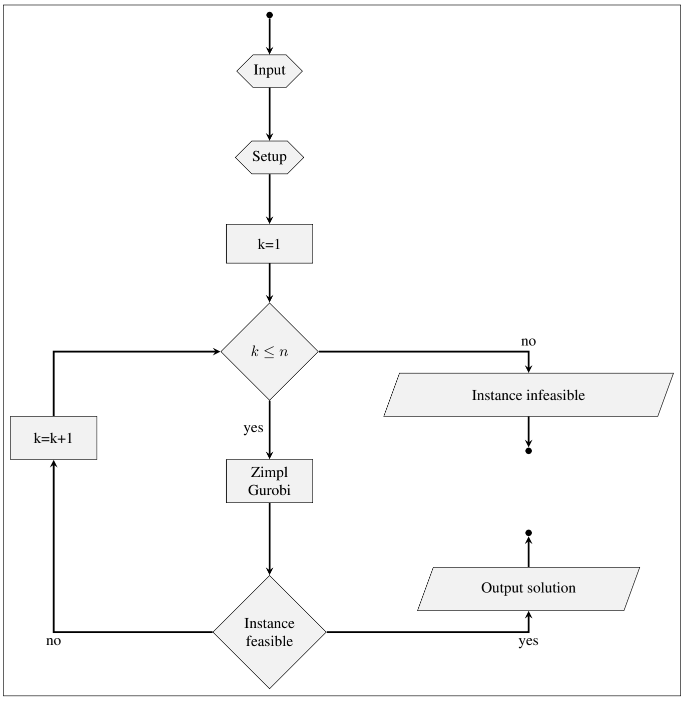


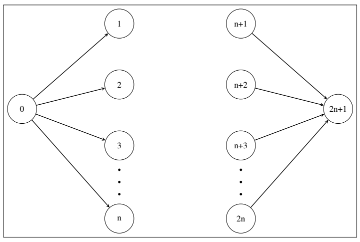
Figure 3: General flow for the model implementation

picked for each vertex i in P. This number represents the number of passengers which share the same request at approximately the same time.

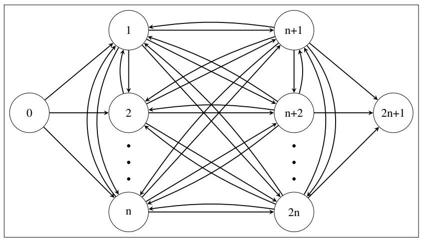
The arc creation is another crucial step in the setup phase. In the original DAR model from Cordeau and Laporte (2003), the arcs were created as follows. At first, the vertices $0, 1, \ldots, 2n + 1$ are initialised. Then, the depot connections from 0 to P and D to $2n+1$ are set up for all vertex pairs. The arcs induced by the vertices from P and D form a complete graph K_{2n}. A bonus arc $(0, 2n + 1)$ can be introduced for routing spare trains if the number of trains is to be minimised as an (secondary) objective.

Now, with the extended model, intermediate vertices have to be added to keep track of the train positions and to determine the occupation of track sections by means of headway times. Each arc in the K_{2n} is removed and replaced by n_{stat} vertices which are then connected according to the corresponding request pattern. Usually, the physical infrastructure graphs for rural environments are circle-free and therefore, the shortest-path between pick-up and drop-off vertex is chosen for the scheduling. Depending on the travel direction of the trains, the parameter v_{ij}^k is determined and set to -1 for inbound, 1 for outbound and 0 for no travel on the arc (i, j) in the blown up K_{2n}. An illustration of the steps can be found in Fig. 4.

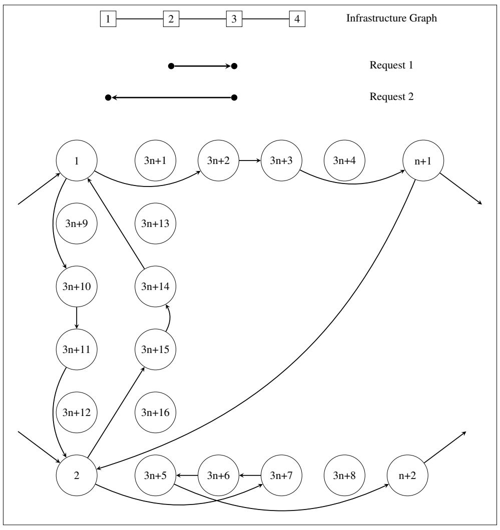
The setup is completed by reading in or randomly generating the costs c_{ij} and travel times t_{ij} for each arc (i, j) as well as setting the dwell/service times d_i and starting and ending time windows e_i and l_i for each vertex $i \in V$. Completing, the lower and upper bounds for the capacity in Eq. (18) need to be computed and the auxiliary variables M_{ij}^k,



(a) The depot connections are set up.



(b) The intermediate arcs are added.



(c) Some exemplary arcs are presented for two requests. These arcs are again split up and further intermediate vertices representing the stations are added.

Figure 4: Graph setup process

W_{ij}^{k} and $Z_{ij}^{k_1,k_2}$ need to be set sufficiently large such that they act as the usual Exclusive OR (XOR).

After initialising all necessary parameters, the actual procedure begins. The number of available vehicles k strongly effects the model size and hence the solubility of the model. Therefore, it turned out to be computationally more efficient starting with a small k and iteratively increasing it. The model is written according to the ZIMPL (Koch (2005)) guidelines and is automatically converted into a standard lp-file. The lp-file is utilised by Gurobi 9.0.1 (Gurobi Optimization (2020)) which solves the instance and outputs all relevant values, e.g. the objective value, calculation time and the optimality gap (if any). If no feasible solution is found (within a reasonable time limit) while iteratively increasing the number of available vehicles the chances are high that no such solution exists at all for this instance.

3.3 Pre-processing techniques

The problem size is a strong weighting factor, besides the problem structure, determining the calculation time and space. For the DARP-R, there exist different angles of attack to reduce the size of the problem.

First, Cordeau (2006) and Ropke et al. (2007) note that the set of arcs in the problem might be reduced due to different logical constraints or shrinking possibilities. These arcs, if initialised at all, which will never be part of a feasible solution can be safely removed before starting the solver. These are, on the high level, (c.f. Fig. 4(b))

- direct backward arcs, i.e. arcs of the type $(n+i, i)\ \forall\, i \in P$,
- arcs from the starting depot to drop-off points, i.e. arcs of the type $(0, n+i)\ \forall\, i \in P$,
- arcs from the pick-up points to the final depot, i.e. arcs of the type $(i, 2n+1)\ \forall\, i \in P$,
- arcs violating capacity constraints, i.e. arcs (i, j) which are part of the path $i \to j \to n+i$ with $j \neq n+i$ and for which the path exceeds the capacity of the vehicle, i.e. if $q_i + q_j > \max\limits_{k \in K} Q^k$ holds,
- arcs violating ride time constraints, i.e. arcs (i, j) which are part of the path $i \to j \to n+i$ with $j \neq n+i$ and for which the path exceeds the ride time of request i, i.e. if $t_{ij} + d_j + t_{j,n+i} > L$ holds,
- arcs violating time windows, i.e, arcs (i, j) for which $e_i + d_i + t_{ij} > l_j$ holds.

After removing the arcs, there might still be potential for tightening of the time windows according to the initialisation. For all arcs of the type $(i, n+i)\ \forall\, i \in P$ it can be checked whether $l_i + d_i + t_{i,n+i} > l_{n+i}$ holds. If this is the case, then l_i can be reduced to $l_i = l_{n+i} - d_i - t_{i,n+i}$.

Second, all unused vertices are removed, i.e. vertices which have no incident arc. These are a part of the vertices that were created by the replacement of the initial arcs.

Third, ideally the amount of headway constraint pairs is to be reduced as well. However, in contrast to the original employment of this method described by Castillo et al. (2011) it is not that apparent which possible train runs will never meet in the temporal dimension. The reduction of headway constraints in the spatial dimension is ensured by the parameters v_{ij}^k for all arcs $(i, j) \in A$ and $k \in K$.

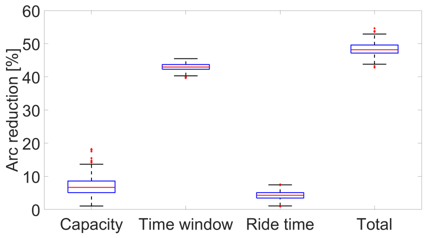
4 Numerical Example

The performance of the model is evaluated through a series of numerical experiments to estimate the effectiveness of pre-processing. The case study considers a small single-tracked line of 22 km length which is currently disused and could be reactivated. It consists of 6 stations and is situated in the heart of Saxony, Germany. The line plan is presented in Fig. 5.

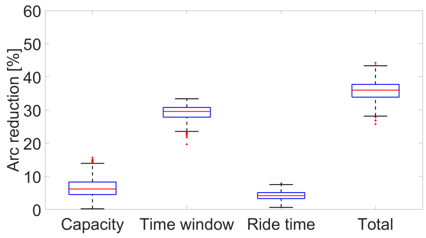
Due to the high number of different parameters, only a small number of the different cases can be examined. For the following study on the effectiveness of pre-processing, the parameters are set as described below. The running time is assumed to be 50 km/h on average such that the travel time between the stations are $t = [5.41, 4.10, 7.61, 5.88, 4.15]$ minutes. The upper time limit for the investigation period and for vehicle usage are no

Figure 5: Line plan of the disused part of the railway line Riesa–Nossen

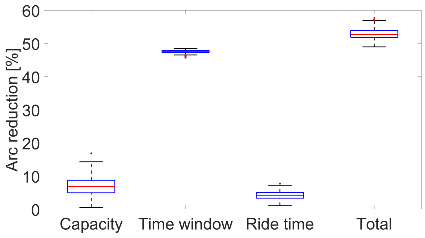
constraining barrier ($\bar{T} = T^k = 600, \forall\, k \in K$). The maximum ride time for each passenger is set to exceed not more than 20% of the maximal ride time ($L = 1.20 \sum t$). $n = 20$ requests are generated and each vehicle has a capacity of $Q^k = 8, \forall\, k \in K$. Finally, the number of passengers for each request is drawn uniformly ($q_i \sim \mathcal{U}(1, 6), \forall\, i \in P$) as well as the dwell/drive-through times ($d_i \sim \mathcal{U}(1, 2), \forall\, i \in H$).



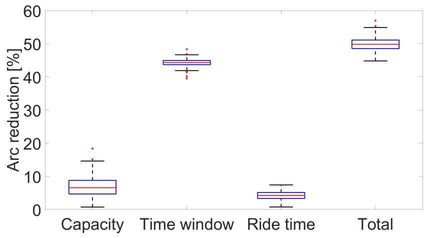
(a) $e \sim \mathcal{U}(1, 120)$, size of time window for pick-up/drop-off: 10/30 minutes



(b) $e \sim \mathcal{U}(1, 120)$, size of time window for pick-up/drop-off: 30/50 minutes



(c) $e \sim \mathcal{U}(1, 560)$, size of time window for pick-up/drop-off: 10/30 minutes



(d) $e \sim \mathcal{U}(1, 560)$, size of time window for pick-up/drop-off: 30/50 minutes

Figure 6: The effectiveness of pre-processing

The results of four different pre-processing scenarios are shown in Fig. 6. Each scenario contains 500 instances that were run according to the aforementioned input parameters. For each scenario the request time window for the pick-up is uniformly drawn either within 120 or 560 minutes. The time window is then computed tight, i.e. a pick-up has to happen within 10 minutes and the drop-off after at most 30 minutes, or loose, i.e. a pick-up has to happen within 30 minutes and the drop-off after at most 50 minutes, Three possible arc removal techniques are studied. First, the capacity, i.e. when the pick-up of a second request would exceed vehicle capacity. Second, arcs can be eliminated if a vehicle cannot be within the allocated time window at the vertex of another request. Third, each passenger has a maximal ride time and further detours for ride-sharing are hence limited. Finally, the total amount of arcs that could be removed is presented.

The contribution of the individual arc reduction methods varies from case to case and can be weighted differently or be more or less. In the expected value, about 40% of the edges in P can be deleted by the capacity criterion. However, since the edges between P and D and within D are not affected, the magnitude agrees very well with the expectation. The time window criterion has by far the greatest influence in this scenario and ensures that up to 50% of the edges can be deleted. If the time windows are larger, more requests can be linked and the proportion of removed edges in Fig. 6(b) and 6(d) is correspondingly lower. In the case of Fig. 6(d), the requests are potentially so far apart that the size of the time windows plays a smaller role. The ride time criterion is currently only globally defined and not related to the individual requests. Therefore, the effect is rather minor and long journeys in opposite directions are eliminated rather than ride-sharing being prevented.

All in all, it can be concluded that pre-processing contributes significantly to better solvability and that many instances cannot be solved without it. Therefore, there is certainly further potential for optimisation here and presumably more conditions can be set up for arc elimination. The size of the problem instance does not directly indicate the gain in computational time or even the general solvability, but the question arises whether the solver could identify the arcs as irrelevant just as quickly. The reduction of the edges in pre-processing can be done in a few seconds. In general, the rapidly growing number of constraints is a big problem and ZIMPL (Koch (2005)) could partly create the lp files only with great effort and these are then also correspondingly large.

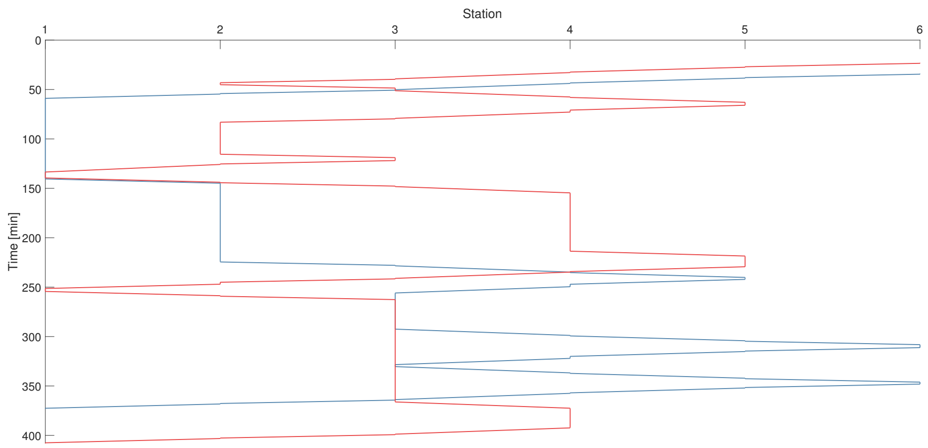


Figure 7: Exemplary time-distance diagram for $n = 12$ requests

Fig. 7 shows an example of the time-distance graph for an instance with 12 requests over a period of about 7 hours. The optimal solution requires 2 trains. Between the requests, the trains dwell in the stations. It can be seen very well, how at minute 140 between the first two stations the following headway times constraint takes effect. The crossing of the trains also works, as can be seen around minute 50 in station 3.

5 Conclusions

The paper has introduced a novel transport concept. DRT on rural railway lines could be a way to keep such lines in operation, especially during off-peak hours, or even reactivate them in the future. Users can send requests to the operator that include pick-up and drop-off location and time windows. The contribution of this paper is the formulation of an optimisation model that creates an optimal schedule for the requests received.

Several challenges arise in the process. Particularly computationally intensive, but important for rural railway lines, are the headway time constraints. Especially in rural areas, the lines are often only single-tracked, so that a crossing is necessary on a regular basis. In general, the many dependencies in the railway system significantly increase the difficulty obtaining a solution.

These difficulties cause that the problem can only be solved optimally for smaller instances. Therefore, the development of sophisticated algorithms or heuristics is inevitable if the methodology is to be available for practical applications. Zhou and Zhong (2007) have calculated better lower bounds to support the optimiser and introduced priority rules. Likewise, Castillo et al. (2009) develop a bisection method which allows for important savings in computational effort. The development of methods to support the optimiser are also a logical further step for DARP-R.

The alternative is to abandon the optimal solution and accept a good approximate solution. For this, different types of heuristics are available, which would have to be combined for the DARP-R. An insertion heuristic could be used to assign passengers and vehicles to possible routings. This can then be supported by a neighbourhood search to further improve the quality of the solution.

Methods to improve the computational speed are necessary before further features can be added in the form of constraints. This could be, for example, a limitation of the individual vehicle operating times depending on their use, if they have to be electrically charged in a depot, for example. Furthermore, the station capacity has not been considered so far and it has been assumed that there are always 2 tracks available and that these are sufficient to park or, if necessary, shunt the vehicles. The formulation incorporates the essential constraints for the operation of small automated rail vehicles on regional lines, but many other cases remain thinkable and open that further complicate the problem.

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