THE IMPACT OF BUFFER TIME DISTRIBUTIONS ON THE NOMINAL CAPACITY OF RAILWAY LINES

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ABSTRACT

Buffer times are essential for preventing delay propagation and ensuring robustness in railway timetabling. While robustness analysis deals with ensuring the effectiveness of allocated buffer times in schedules, the number of trains is generally assumed to be fixed. The feasibility of the train operation concept needs to be checked by strategic long-term capacity planning beforehand. Capacity analysis methods depend on buffer times and involve some sort of delay prognosis. The goal of the present paper is to analyse the effects of buffer time distributions on nominal capacity obtained with stochastic (analytic) capacity analysis approaches. Complementing previous work where Monte-Carlo simulation had been applied, it is shown how convolution integrals arising in analytic delay propagation models can be explicitly calculated using moment generating functions. Based on this approach, a generalisation of the STRELE framework, which is the standard methodology of German infrastructure manager DB Netz AG for capacity analysis of railway lines, is derived and the effects of different buffer time distributions on nominal capacity are studied.

Keywords: railway operations, buffer time, knock-on delay, analytical calculation, nominal capacity.

1 INTRODUCTION

Buffer times play an essential role in railway timetabling. By including spare times on top of technically minimal feasible headway times, the formation of knock-on delays can be impeded and the robustness of the timetable against disturbances is increased [2]–[5].

While buffer times ensure the quality of service in railway operations they bear on the usable capacity in line and frequency planning. In this sense, planning and operations pose contradictory requirements on buffer time allocation. As a result, a better understanding of scheduled buffer times, their effectiveness and their effects on rail capacity is of vital interest in long-term strategic planning.

Whereas tactical planning of capacity is mostly based on timetable-based methods such as UIC code 406 [6], long-term strategic planning has to cope with the difficulty that the timetable is subject to major changes or may not be known at all. This is why stochastic delay prognosis modelling is prevailing in this area. Many models rely on queueing methods [7]–[10], where buffer times are modelled implicitly in the arrival process. Another class of models is based on convolutions of probability density functions to calculate (knock-on) delays: in [2] a family of θ-exponential polynomials are used to represent delay distributions. Meester and Muns [11] use a similar concept based on phase-type distributions has been discussed. A general approach relying on numerical evaluation of the integrals is discussed by [12].

In the present paper the focus is on a specific approach, the STRELE framework, which is the standard approach of German infrastructure manager DB Netz AG for the capacity analysis of railway lines [13]. The foundations of the approach, which is a part of a family of infrastructure-centred stochastic capacity analysis tools used by DB Netz AG [13] have originally developed by Schwanhäußer [3]. It provides a purely analytic method to calculate the mean knock-on delays based on (train-specific) probability distributions of primary delays and buffer times between trains. The approach has recently been revisited and formalised in [15].

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An important aspect of Schwanhäußer's original derivation of the pivotal STRELE formula, an analytic approximation of the average height of knock-on delays, is the assumption that buffer times are either fix or exponentially distributed random variables [3]. This assumption seems questionable nowadays, given the fact that timetables tend to get more and more periodic [16]. We show how the formal description given in [15] can be used to remedy this shortcoming by allowing to derive train-specific approximations for the height of knock-on delays for any buffer time distribution which allows for a closed form representation of the moment generating function. This class of distribution functions include normal distributions, phase-type distribution functions as well as all distribution functions, which allow for a matrix exponential representation of the pdf. As a result, a wide range of buffer time distributions and their effects on knock-on delays, and hence line capacity, can be modelled.

The current presentation builds on previous work, where the authors have analysed buffer time distributions in delay propagation modelling using Monte Carlo simulations [1]. The present paper complements [1] by showing how a generalization of buffer times can be achieved in analytic stochastic modelling approaches. We subsequently present an enhancement of the STRELE-formula [3] which allows for flexible buffer time distributions. We analyse how this affects line capacity using knock-on delays as a quality metric. For the admissible delay the level of service used by German infrastructure manager DB Netz AG is applied [13]. While the STRELE procedure is a capacity analysis technique currently predominantly used in Germany, it is transferable to any other infrastructure manager. What is more, similar model logics have been applied in stochastic delay propagation models such as [2], [12]. The techniques based on moment-generating functions described in this paper are transferable and facilitate calculus of delay propagation integrals, in general.

The paper is structured in the following way: we start by giving a short overview on the STRELE-framework in the following section. In particular, we demonstrate how the laborious calculation of the convolution integrals in delay propagation modelling can be greatly facilitated by resorting to moment generating functions, for which closed-form representations can often be given. In Section 3, we discuss the performance of our new approach. Therefore, we confer the results of old and new framework under comparable constraints. Furthermore, we investigate the impact of different buffer time distributions on the nominal capacity of railway lines and get an indication of the number of cases in which the STRELE-formula over- or underestimates the capacity.

2 METHOD

We present the methodology used in this paper in the follow-up. At first, we introduce the STRELE-formula, which has been used for decades to calculate the expected knock-on delays on railway lines. Afterwards, we enhance the existing framework towards modern constraints and requirements. We conclude with a short technical chapter necessary to handle a wide class of buffer time distributions in the model.

2.1 STRELE-framework

The STRELE-framework provides an analytic approximate analysis of the expected knock-on delays on railway lines. It relies on a queueing-theoretic representation of railway lines, where railway line sections are represented by two unidirectional servers (for double-track railway lines), adjacent stations correspond to waiting areas and service times are defined by minimum headway times between subsequent trains [7]. Based on train-specific probability density functions of primary delays conflict, probability and height of transferred knock-on delays are calculated as a function of buffer times between pairs of trains [3]. Here, primary delays are represented by a distribution which consists of a mixture of binomial (delay yes/no) and Exponential distributions (height of delays), which was largely confirmed by comparison to train records [17].

Currently, the highly technical delay propagation modelling in the STRELE framework is condensed in the STRELE formula. Based on the assumption of independently identically exponentially distributed buffer times (alternatively, deterministic buffer times have also been discussed [3]), the mean knock-on delay (per train) is given explicitly by this formula:

$$K = \left(\bar{c} - \frac{\bar{c}^2}{2}\right) \cdot \frac{\bar{t}^2}{\bar{b} + \bar{t}\left(1 - e^{-\frac{\bar{h}}{\bar{t}}}\right)} \cdot \begin{bmatrix} p_{eq}\left(1 - e^{-\frac{\bar{h}_{eq}}{\bar{t}}}\right)^2 \\ +\left(1 - p_{eq}\right) \cdot \frac{\bar{h}_{diff}}{\bar{t}} \cdot \left(1 - e^{-\frac{2\bar{h}_{diff}}{\bar{t}}}\right) \\ +\frac{\bar{h}}{\bar{b}} \cdot \left(1 - e^{-\frac{\bar{h}}{\bar{t}}}\right)^2 \end{bmatrix}, \tag{1}$$

where:

$\bar{c}$	average probability of primary delays
$\bar{t}$	average time of delay of the delayed trains
$\bar{b}$	average buffer time
p_{eq}	probability of trains with equal rank
$\bar{h}$	average minimum headway time
$\bar{h}_{eq}$	average minimum headway time between trains with equal rank
$\bar{h}_{diff}$	average minimum headway time between trains with different rank

The STRELE formula has been implemented in various software tools [18], [19] and is broadly used to compute the expected knock-on delay K. Applying a level of service as an allowed sum of knock-on delays, the capacity of the investigated railway line can be derived. The closed form approximation, however, is obtained by averaging primary delays of trains; hence, only averages are needed as input [3]. This, as well as the i.i.d. assumption of buffer times between trains, gives rise to implausible conclusions: two schedules, for instance, which have the same average buffer time, but largely different distribution of buffer times will result in the same expected knock-on delay.

2.2 Enhanced analytical calculation of the expected knock-on delays

Schwanhäußer's model, his assumptions and the derivation of the STRELE formula have recently been discussed and formalized by Weik et al. [15]. We subsequently present two major extensions of the STRELE framework, which allow to consider and efficiently calculate approximations for a wide class of buffer time distributions and, at the same time, allow as to keep the train-specific information and to avoid resorting to train averages.

As pointed out by Weik et al. [15] the expected waiting times K (knock-on delays) can be split into delays of first and higher order. While the higher order delays can be treated based on a $M/GI/1/\infty$-queueing representation with FCFS service policy (neglecting train priorities in heavy traffic), first-order delays are treated explicitly (potentially including delays) on a distributional basis. A functional relation between total knock-on delays, first-order and higher order delays is built on the upscaling factor ξ:

$$K = K^{(1)} + \frac{1}{\xi} K_{eq}^{(1)}, \tag{2}$$

with

$$K^{(1)} = \sum_{n=2}^{\infty} \left(p_{eq} B_{1n,g} + \left(1 - p_{eq}\right) B_{1n,diff}\right) \cdot e^{-\lambda_1 h_{1n}} \int_0^{\infty} e^{-\lambda_1 b_{1n}} \, dF_{\hat{b}_{1n}}(b_{1n}). \tag{3}$$

To calculate $K_{eq}^{(1)}$ the probability of trains with equal rank p_{eq} is set to one. For the technical details of the calculation of ξ and the parameters in equation (3) the reader is referred to [15].

The computation of the expected knock-on delays (first or higher order) are dependent on three components:

- The sum corresponds to conflicts of the observed train with the, theoretically infinitely many, following trains. In general, the summands are getting smaller than machine precision after a few trains.
- The factor in front of the integral is only dependent on train characteristics like minimum headway times and delay characteristics.
- The integral itself is only dependent on the buffer time distribution in the schedule and will be examined more closely in the following subsection.

This intermediate formula $K^{(1)}$ in [15] contains the entire train-specific information including sequence-dependent headway times, train-specific parameters of primary delays. Thus, building on this formula, we only have to discover the means to efficiently evaluate eqn (3) for more general buffer time distribution present in the schedule.

2.3 Moment-generating functions

Having a closer look on (3), it can be seen that the buffer time distributions only enter in the integral on the right-hand-side. In case it exists, this integral can be represented by the moment-generating function (mgf),

$$M_{\hat{b}_{1n}}(-\lambda) = \int_{-\infty}^{\infty} e^{-\lambda b_{1n}} dF(b_{1n}), \tag{4}$$

of the probability distribution of the buffer times, where F is the corresponding cdf. The mgf M_X is an alternative definition of a random variable's X probability distribution.

It turns out that for a large class of probability distributions, including Erlang distributions, Gamma distributions, Degenerated distributions or Chi-Squared distributions a closed form expression of the mgf can be given [20]. The laborious task of numerically integrating the buffer time integral for each pair of trains is hence reduced to simple function evaluations. Particularly remarkable is the fact that the Degenerate (Dirac) distribution can be represented in this way, which means the approach can also be adopted to represent exact (scheduled) buffer times.

3 RESULTS

To present the impact of different buffer time distributions and input parameters, we examine the influence of the aforementioned on the nominal capacity for 1008 scenarios. At first, we verify the new model on a scenario where both, the STRELE-formula and the enhanced framework, are comparable. Following, we present the reader the procedure of the calculation of the nominal capacity for one scenario. In the parameter study, the results of the set of scenarios are evaluated and finally we conclude with a short note on the computational complexity of the enhanced framework.

3.1 Consistency of the enhanced framework

The enhanced STRELE-framework is an extension of the classical STRELE-formula. Therefore, new and previous formula should be consistent, in the sense that they produce the same knock-on delays for comparable input. Indeed, as it can be seen in Fig. 1, the two results are equivalent as long as the input parameters are variance-free and exponentially distributed buffer times are assumed. If the input parameters deviate, the resulting knock-on delays differ due to the restriction of averaged parameters in the old STRELE-formula. The effects of varying parameters are discussed in more detail in Section 3.3.

3.2 Influence of buffer time distributions

Given a specific scenario, i.e. minimum headway times, delay characteristics, buffer times, et cetera for all trains, one is interested in calculating the quality of the schedule. In Germany, directive 405 of DB Netz AG [13] defines the level of service based on the acceptable height of knock-on delays. This Level of Service (LOS) depends on the type of train services and is calculated based on the following formula

$$LOS = 0.257 \cdot exp(-1.3 \cdot p_{pt}) \cdot t_{sched}, \tag{5}$$

where p_{pt} is the share of passenger trains and t_{sched} the length of the investigation period.

Knowing the acceptable height of knock-on delays, it is possible to determine the average minimum required buffer time for each scenario. Exemplary, in Fig. 2 the calculated knock-on delays for different buffer time distributions and the STRELE-formula in dependence of the average buffer time are depicted. Graphically intersection the LOS and the approximation formula for the height of knock on delays the optimal height of buffer times (per train), on

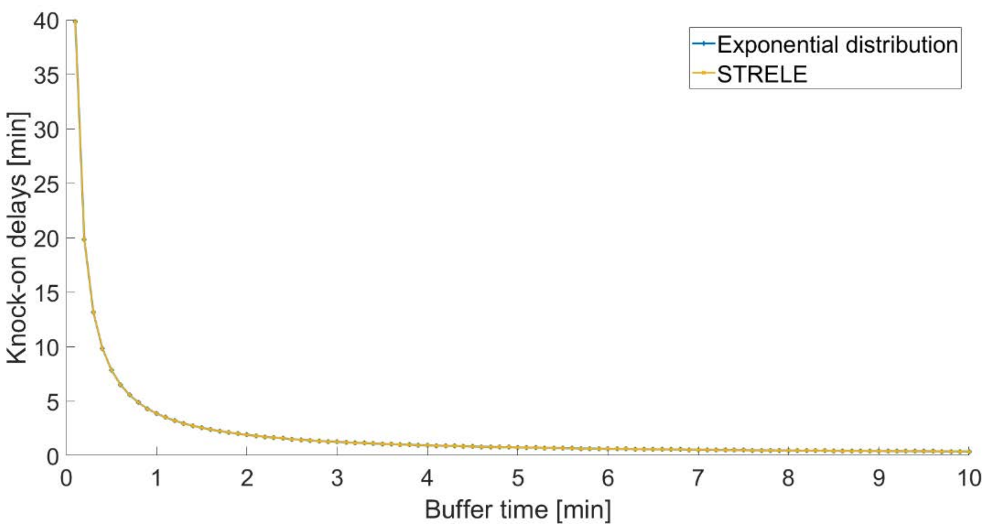

Figure 1: Consistency of the enhanced STRELE framework for non-train specific primary delays and exponentially distributed buffer times.

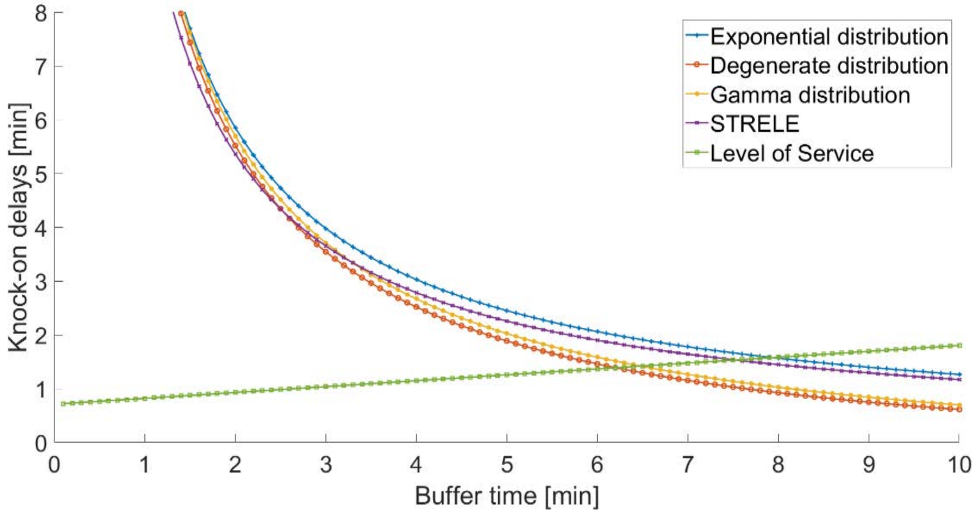

Figure 2: Calculation of the required average minimum buffer time for the scenario given the necessary service quality and depending the buffer time distribution.

average, can be obtained. This point refers to the minimum buffer time that ensures satisfactory quality of service in operations. As it can be seen, the minimum buffer time required differs depending on the observed buffer time distribution and the scenario.

Knowing the average minimum headway time $\bar{h}$ from the scenario input and reading off the minimum buffer time b_{min}, the nominal capacity N_{opt} can be calculated for every time frame t_{sched} by

$$N_{opt} = \frac{t_{sched}}{\bar{h}+b_{min}}. \quad (6)$$

3.3 Parameter study

In the following, a set of 1008 scenarios is investigated. The analysis consists of four different matrices of minimum headway times, six different vectors of delay probability and delay height each and seven different train mixtures. A detailed description of the scenario setup can be found in Tables 1–4. Each scenario contains two long-distance trains (LDT1, LDT2), two local trains (LT1, LT2) and two freight trains (FT1, FT2).
For each scenario the nominal capacity is calculated, as presented in the previous chapter, for Exponential, Gamma (3) and Degenerated distribution and compared to the state-of-the-art nominal capacity given by the STRELE-formula.

The subsequent figures display the relative difference of nominal capacities RD between one of the chosen distributions N_{dist} and the STRELE-formula N_{STR} as well as the absolute difference of nominal capacities AD, which are calculated as follows:

$$RD = \frac{N_{dist}-N_{STR}}{N_{STR}}; AD = N_{dist} - N_{STR}. \quad (7)$$

The figures show a histogram in which the number of cases for a specific relative difference of the nominal capacity (red), respectively the absolute difference of the nominal capacity (blue) is displayed on the y-axis.

Table 1: Minimum headway times (min).

Scenario 1	LDT1	LDT2	LT1	LT2	FT1	FT2
LDT1	4	4	4	4	4	4
LDT2	4	4	4	4	4	4
LT1	4	4	4	4	4	4
LT2	4	4	4	4	4	4
FT1	4	4	4	4	4	4
FT2	4	4	4	4	4	4
Scenario 2	LDT1	LDT2	LT1	LT2	FT1	FT2
LDT1	3.77	6.96	4.14	3.17	5.83	5.83
LDT2	2.10	4.96	5.92	2.75	5.93	5.93
LT1	3.93	3.33	3.26	2.68	2.02	2.02
LT2	7.60	8.91	9.17	4.84	8.40	8.40
FT1	5.07	6.89	7.48	3.89	3.78	3.78
FT2	5.07	6.89	7.48	3.89	3.78	3.78
Scenario 3	LDT1	LDT2	LT1	LT2	FT1	FT2
LDT1	3.13	2.73	4.17	4.32	5.88	5.88
LDT2	6.21	3.13	4.18	4.32	5.88	5.88
LT1	5.32	4.20	3.40	4.27	1.99	1.51
LT2	6.96	6.96	6.98	4.27	8.33	8.33
FT1	5.98	5.98	7.66	3.73	3.73	4.04
FT2	5.91	5.91	8.12	3.73	3.42	3.73
Scenario 4	LDT1	LDT2	LT1	LT2	FT1	FT2
LDT1	3	4.4	6	8	10	13
LDT2	3	4	5.5	7.3	10	13
LT1	3	4	5	6.2	8	12
LT2	3	4	5	6.2	8	11
FT1	3	4	5	5.8	7.6	11
FT2	3	4	5	5.8	7.6	9.3

Table 2: Probability of being delayed.

	LDT1	LDT2	LT1	LT2	FT1	FT2
Scenario 1	0.5	0.5	0.5	0.5	0.4	0.4
Scenario 2	0.5	0.5	0.5	0.5	0.5	0.5
Scenario 3	0.1	0.2	0.4	0.6	0.8	0.9
Scenario 4	0.3	0.3	0.5	0.5	0.7	0.7
Scenario 5	0.2	0.2	0.2	0.2	0.2	0.2
Scenario 6	0.8	0.8	0.8	0.8	0.8	0.8

Table 3: Average height of delay for the delayed trains (min).

	LDT1	LDT2	LT1	LT2	FT1	FT2
Scenario 1	5	5	2	2	20	20
Scenario 2	5	5	5	5	5	5
Scenario 3	1	2	4	6	8	9
Scenario 4	2	2	2	2	2	2
Scenario 5	7	7	7	7	7	7
Scenario 6	5	5	8	8	12	15

Table 4: Train mixture.

	LDT1	LDT2	LT1	LT2	FT1	FT2
Scenario 1	1/6	1/6	1/6	1/6	1/6	1/6
Scenario 2	0.5	0.5	0	0	0	0
Scenario 3	0.15	0.15	0.35	0.35	0	0
Scenario 4	0	0	0.3	0.3	0.2	0.2
Scenario 5	0	0	0	0	0.6	0.4
Scenario 6	0	0	0	1	0	0
Scenario 7	0.1	0.15	0.3	0.25	0.1	0.1

According to Directive 405 of DB Netz [13] a five-hour time-frame has been taking for the examination of the influence of the selected buffer time distributions compared to the current calculation method.

One of the main assumptions in the derivation of the STRELE-formula are exponentially distributed buffer times. As it can be seen from Fig. 3, the difference between the STRELE-formula and the new framework with exponentially distributed buffer times is very small for the observed scenarios. As one might expect the assumption of small minimum headway times variance and taking the average in the original STRELE-formula is not contemporary on the one hand and seems to generally overestimate the expected knock-on delays on the other hand for exponentially distributed buffer times.

Around half of the scenarios show negligible differences in the nominal capacity leading to an average of 3.2% (0.88 trains) less capacity in a 5-hour period. In two extreme cases, the STRELE-formula underestimates the capacity by 4.33 trains and overestimates by 0.97 trains.

The Gamma distribution is an expansion of the Exponential distribution and allows covering more complex buffer time structures in the schedule. We observed the results of a Gamma (3) distribution, which can be depicted in Fig. 4, and its behaviour differs from the previous result. The STRELE-formula could not capture such schedule structures leading to an underestimation of an average 4.8% (1.46 trains) in nominal capacity. In contrast to the STRELE results, the infrastructure manager would be able to operate up to 5.84 trains more respectively operates up to 2.90 trains too much with respect to the same quality as before in the observed scenarios.

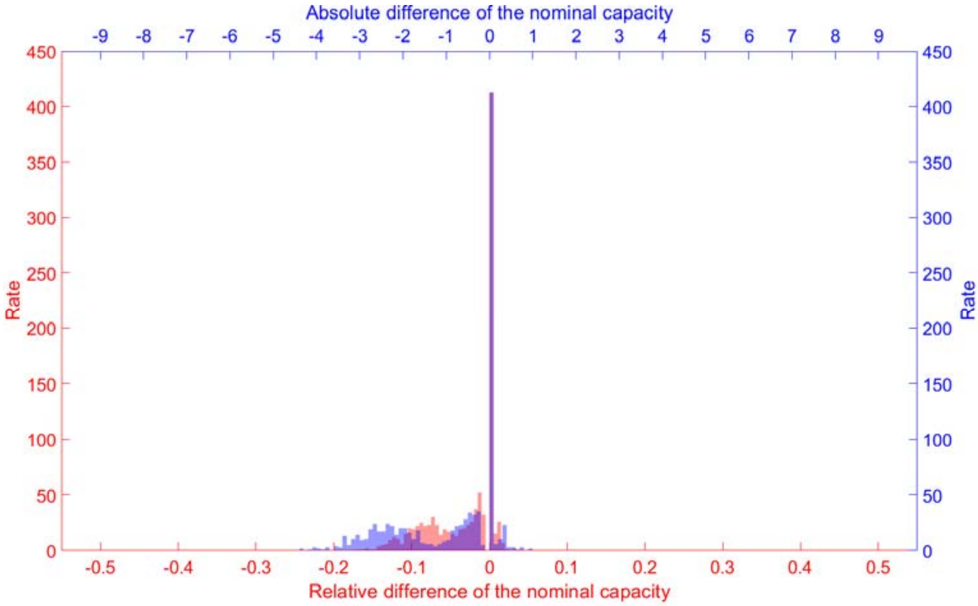

Figure 3: The difference in nominal capacity between the new framework with exponentially distributed buffer times and STRELE-formula.

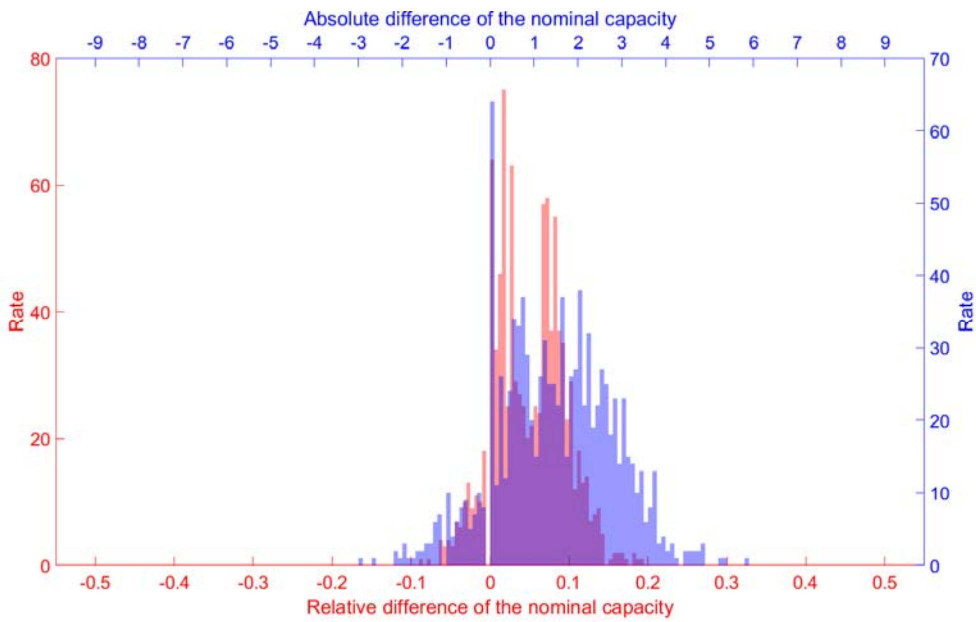

Figure 4: The difference in nominal capacity between the new framework with gamma distributed buffer times and STRELE-formula.

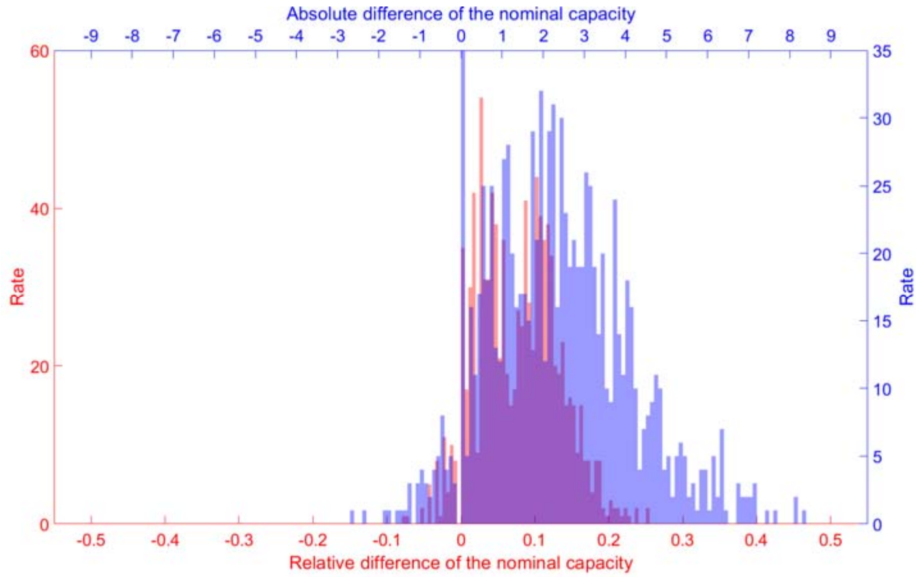

Figure 5: The difference in nominal capacity between the new framework with degenerated distributed buffer times and STRELE-formula.

The Degenerate distribution fits best for periodic schedules where fitting a continuous distribution with decent quality fails. Therefore, it can be seen as generalisation of all the other distributions representing the exact buffer times in the schedule.

The STRELE-formula can calculate nominal capacities for highly periodic timetables but is not designed to do so. Hence, leading to an average 7.6% (2.35 trains) underestimation of the nominal capacity for the given scenarios. In the extrema, it leads to an overestimation of 2.61 trains and an underestimation of 8.33 trains in the five-hour period.

In summary, the nominal capacity can be computed for a broad class of buffer time distributions. It is now particularly possible to deal with periodic schedules. The results show that it is necessary to incorporate the buffer times in the schedule, to dismiss the assumption of exponentially distributed buffer times and the requirement of similar train characteristics corresponding to close to average minimum headway times.

3.4 Computation time analysis

The calculation of the knock-on delays within the extended STRELE-framework is more complex than the evaluation of the original closed STRELE-formula. For every one of the m trains the expected impact on the following n trains has to be calculated for the first-order and higher order knock-on delays in $O(1)$. In summary this leads to a computational complexity of $O(n \cdot m)$. For real instances the computation time lies within a few seconds such that the determination of the buffer times within the schedule and the calculation of minimum headway times should be the main time consumers in the capacity analysis.

4 CONCLUSION

The current state-of-the-art process to calculate the nominal capacity of a railway line has been presented and extended. It has been shown that a refinement of the STRELE-formula has been necessary due to the critical assumptions of exponentially distributed buffer times and similar minimum headway times. The developed framework is based on the derivation of the STRELE-formula and can be viewed as an enhancement of the existing method enabling infrastructure managers to better assess periodic schedules.

Furthermore, it has been demonstrated in the parameter study that the extension was required. Due to current limitations buffer time distributions different from exponentially distributed could not be considered in detail. The error made was quite significant – leading to over- or underestimations of the nominal capacity of up to 1.67 trains per hour in the observed scenarios. It became clear that the nominal capacity is dominated by the underlying buffer time distribution.

The method is comparably easy to implement and use and can be utilised by infrastructure managers outside of Germany, too.

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